# An Adaptive Energy Detection Scheme Using EMD for Spectrum Sensing

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***Abstract*—Noise estimation plays an essential role in enhancing the performance of non-coherent spectrum sensors such as energy detectors. If the noise energy is misestimated, detector performance may deteriorate. In this paper, we present an energy detector based on the behavior that Empirical Mode Decomposition (EMD) has towards vacant channels (noise-only). EMD decomposes time-series signals into a finite set of components called Intrinsic Mode Functions (IMFs). The energy trend of these IMFs (modes) is used to determine the occupancy of a given channel of interest. The performance of the proposed EMD-based detector is evaluated for different noise levels and sample sizes. Further, a comparison is carried out with conventional spectrum sensing techniques to validate the efficacy of proposed detector and the results revealed that it outperforms the other sensing methods.**

***Index Terms*—Cognitive radio, spectrum sensing, EMD, noise estimation, energy detection.**

## I. INTRODUCTION

Cognitive radio enables the use of radio spectrum that is underutilized due to the fix allocation of frequency bands [1]. In the physical layer of cognitive radio, spectrum sensing plays a key role in detecting the holes (vacant channels) for a given band [2]. Further, energy detection (ED) is used widely for spectrum sensing purposes due to its noncoherent nature and low computational complexity [3].

Energy detectors work by comparing the energy of the channel under test to a predefined threshold to make the the occupancy decision [4]. However, ED threshold is a function of noise energy which is in turn assumed *a priori* or estimated through measurements in nearby channels. Consequently, misestimating the noise energy might result in a severe degradation in the detector performance [5]. In practical scenarios, noise energy can vary over wireless channels and due to the thermal noise at the receiver front-end leading to what is called noise uncertainity [6].

More recently, Empirical Mode Decomposition (EMD) has been proposed as a detection method for wireless applications [7]. EMD is an adaptive and blind technique that decomposes time-series signals into a set of modes called Intrinsic Mode Functions (IMFs) [8]. Similar to ED, EMD behaves non coherently towards the received signals in which it requires no prior information about the signal charactersitics.

Roy and Doherty use EMD, in general, to enhance the detection of weak signals in the presence of noise [9]. The detection method in [9] requires numerous calculations that may not make it practical for real-time sensing. In an effort to detect non-stationary and non-linear signals, Bektas *et al* proposed a spectrum sensing algorithm using relative entropy [10]. However, this method also requires a large number of calculations to determine the classifier for separating a signal from noise. In [11], a non-parametric threshold is derived from a set of IMF powers in the frequency domain and used for multi-channel detection. However, the determination of the candidate channel is obtained through a heuristic approach without the use of a designed probability of false alarm which makes the proposed method lack for generality.

In this paper, the monotonic decreasing trend for a set of IMF energies will be used to decide the vacancy of a given channel for a designed probability of false alarm. The proposed method is blind and therefore does not require a knowledge of the received signal characterstics to make a detection decision. Further, it is robust to noise uncertainty as the used threshold is not a function of the noise energy. In Sec. II, we provide background on the EMD operation and a description of the system model. Details of the proposed method are presented in Sec. III. In Sec. IV, we provide simulation results and conclude our work in Sec. V.

## II. THEORETICAL BACKGROUND

In this section, the theoretical background of EMD and its characteristics are presented. Further, a formulation of the spectrum sensing system model and its null hypothesis metric are outlined.

### *A. Empirical Mode Decomposition*

EMD decomposes time-series signals into a complete and finite set of oscillatory basis functions that are known as Intrinsic Mode Functions (IMFs). These functions must satisfy the following conditions: 1) the number of extrema and the number of zero-crossings must be either equal or differ at most by one for the whole data set and 2) the mean of the local maxima/minima envelopes is zero at any point.

The modes (IMFs) are collected through an iterative process called sifting. The sifting process eliminates most of the signal anomalies and makes the signal wave profile more symmetric. EMD works as a dyadic filter banks matching the behavior of wavelet transform in terms of decomposition nature [8], [12]. Therefore, the frequency content embedded in the processed modes reflects the physical meaning of the underlying frequencies. The EMD algorithm can be outlined as follows [8]:

1) Identify all extrema points (local maxima and minima) of input signal $y(n)$ and interpolate them (cubic spline interpolation) to find the upper and lower envelopes $e_{\max}(n)$ and $e_{\min}(n)$ respectively.
2) Find the local mean: $m(n) = \big(e_{\min}(n) + e_{\max}(n)\big)/2$.
3) Extract the detailed signal: $h(n) = y(n) - m(n)$.
4) If $h(n)$ does not satisfy the stoppage criteria[1], then the process is repeated and $h(n)$ is the input to step (1). Otherwise, $h(n)$ is the $i^{\text{th}}$ IMF and the residue is $r(n) = y(n) - h(n)$ will be processed as input signal (steps 1-4).

The original input signal can be reconstructed as follows:

$$\hat{y}(n) = \Re(n) + \sum_{i=1}^{\mathrm{K}} \mathcal{M}_i(n) \tag{1}$$

where $\hat{y}(n)$ is the reconstructed signal, $n$ is the sample index, K is the total number of IMFs, $\mathcal{M}_i(n)$ is the $i^{\text{th}}$ mode (IMF), and $\Re(n)$ is the trend of $\hat{y}(n)$.

One characteristic of EMD is that sum of all IMFs $\big($See eq. (1)$\big)$ results in the original signal, which shows the additive reconstructive nature of IMFs in $\ell1$-norm sense. The IMFs are also additive in the $\ell2$-norm sense $\big($the sum of IMFs powers approximates the power of the processed signal, $y(n)\big)$ and resemble the outputs of a dyadic filter bank [12]. Also, the sample size plays an essential role in the behavior of the EMD sifting performance as EMD was designed originally to process continuous signals. Therefore, oversampling is required to capture all possible extrema, whereas, missing an extrema during the sifting process might result in losing an oscillation (or produce false envelopes) and hence the reconstructed signal, $\hat{y}(n)$ will not represent the original signal [13]. Conversely, uncorrelated noise samples will not be influenced by missing extrema as the resulting envelopes will reflect the highest frequency content and thus get sifted largely by the first few modes.

### B. System Model

For a channel under sensing, the received signal $y(n)$ either contains only noise samples $w(n)$, or occupied by a primary user (PU) and contains signal $s(n)$ pluse noise $w(n)$ samples. The detection problem can be formulated (as shown below) into a binary hypothesis test where, $\mathcal{H}_0$ and $\mathcal{H}_1$ represent the absence and the presence of user activity respectively, such that:

$$y(n) = \begin{cases} w(n) & : \mathcal{H}_0 \\ s(n) + w(n) & : \mathcal{H}_1 \end{cases} \tag{2}$$

where $n = 1 \ldots N$ and $N$ is the sample size, while, $y(n)$, $s(n)$, and $w(n)$, are the received signal, transmitted signal, and white Gaussian noise (wGn), respectively.

[1]Cauchy convergence test is used in this paper as a stoppage criteria with a stopping value of 0.2 [8].

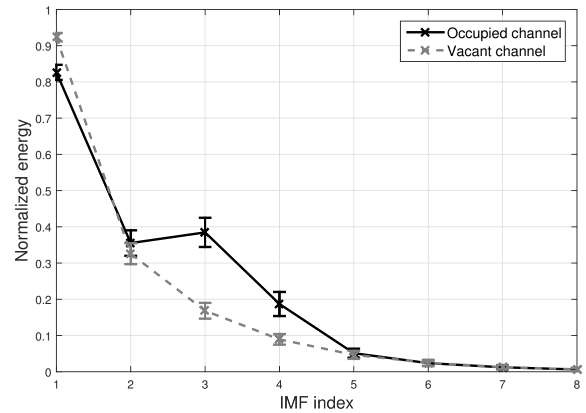



Figure 1: A comparison of IMF energies for an occupied and vacant channels

## III. Proposed Scheme

In this section, an EMD-based spectrum sensing approach is presented. The basic idea is that if the IMF energies deviate from the monotonically decreasing trend, this will result in the decision of occupancy.

### A. EMD Behavior Towards Vacant and Occupied Channels

In Sec. II-A, it was shown that the IMFs resemble the output charactersitcs of dyadic filter. For time-series signals contaminated with Gaussian noise, the EMD sifting process extracts the noise components (fast oscillations) in the lower IMF indices and the signal components (slow oscillations) in the higher ones. The energy of these IMFs reflects the contributions of both noise and signal components to the total energy of the input noisy signal. Herein, if the noise contribution, of the input noisy signal, was much higher than the signal, or the noise-only signal is presented, then the IMF energies will follow a decreasing trend (exponential decay).

To understand this characteristic more clearly, we present Fig. 1 that shows how the IMF energies behave when a received signal is noise-only (vacant channel) or noisy signal (occupied channel) at SNR $= -5$ dB. From this figure, it is shown that the energy of IMF$_3$ (in black solid line) deviates from the vacant channel model (in grey dashed line). The justification of such deviation is due to the presence of PU, where the contribution of signal is added to the noise leading to non decreasing trend of the IMF energies and hence provide an indication of occupancy. The monotonic decreasing trend of IMF energies is exploited in this paper to set a null hypothesis test for spectrum sensing purposes.

### B. EMD-based Detection Scheme

The received signal $y(n)$ is the time-series representation of a given channel under test. Applying EMD approach on $y(n)$ will result in a set of IMFs (see Sec. II-A). The energy of the $i^{\text{th}}$ IMF is given as follows:

$$\mathbb{W}_i = \sum_{n=1}^{N} \mathcal{M}_i^2(n) \tag{3}$$

The detection decision is made upon simple test of the IMF energies given in eq. (3), in which the channel vacancy is decided if the following condition is met:

$$\log_2(\mathbb{W}_i) > \log_2(\mathbb{W}_{i+1}) \quad \forall i = 1 \dots K-1 \tag{4}$$

Consequently, if the condition in (4) is not satisfied, a possible channel occupancy is declared. However, due to the non-linear filtering nature of EMD, relying on the condition (4) might result in false detection especially in low SNR regimes.

To cope with the problem of false detection, an energy model with confidence interval limit can be utilized based on the work in [14]. Ideally, the energy model with confidence interval is based on the noise-only model, which requires the knowledge of the received signal's noise. The noise-only energy model of the $i^{th}$ IMF can be given as follows:

$$\hat{\mathbb{W}}_i = \mathbf{W} \times \mathbf{2}^{-i} \quad i \geq 2 \tag{5}$$

where $\hat{\mathbb{W}}_i$ is the $i^{th}$ estimated noise-only IMF energy, $\mathbf{W}$ is the energy of the total noise in the received signal, and the based $\mathbf{2}$ exponent refers to the energy decreasing rate of IMFs conducting the findings that EMD behaves like a dyadic filter [12]. Practically, the model in (5) requires a knowledge of noise energy ($\mathbf{W}$), which is unknown, hence estimating the noise energy plays an essential role in this respect.

In [14], for a noisy signal, the first IMF, $\mathcal{M}_1(n)$, is assumed to be dominated mostly by noise, hence a noise-only energy model based on the use of $\mathcal{M}_1(n)$ can be utilized to set a statistical boundary limit. The noise-only energy model is given as follows:

$$\hat{\mathbb{W}}_i = (\sqrt{\rho} \times \mathbb{W}_1) \times \mathbf{2}^{-i} \quad i \geq 2 \tag{6}$$

where $\mathbb{W}_1$ is the energy of the first IMF, and $\sqrt{\rho}$ is a scaling factor[2]. In eq. (6), the term $(\sqrt{\rho} \times \mathbb{W}_1)$ is the estimation of the received noise energy. However, in [14], the scaling factor ($\sqrt{\rho}$) is fixed scale thus, the noise estimation will be a function of the first IMF energy only.

In this paper, we propose an adaptive scaling factor that can result in better estimation of the received signal's noise energy. The proposed scaling factor is based on the ratio of the $\text{IMF}_1$ energy to the total noise energy of the received signal and that ratio is denoted by $\beta$ such that:

$$\beta = \frac{\mathbb{W}_1}{\mathbf{W}} \tag{7}$$

Unlike the fixed scale ($\sqrt{\rho}$), the proposed scale is a function of the received signal sample size ($N$). Fig. 2 illustrates the relationship between the sample size (base 2) and $\beta$. From

[2]For wGn time-series, the experiments in [14] showed that $\rho \sim \frac{\mathcal{Z}(\mathcal{M}_i(n))}{\mathcal{Z}(\mathcal{M}_{i+1}(n))} \sim 2$, where $\mathcal{Z}$ is the number of zero-crossings.

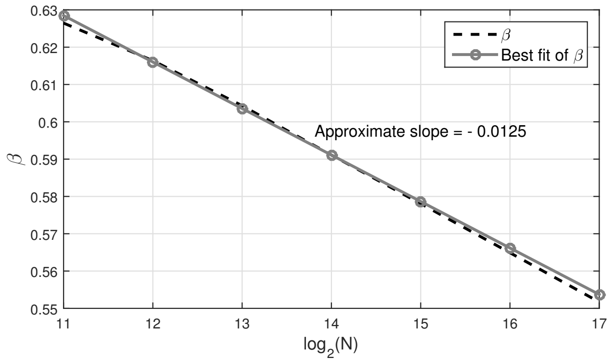



Figure 2: The relationship between the number of samples ($\log_2(N)$) of the first IMF and $\beta$ (dashed line) compared to the best fit model (solid line) using least-squares fit polynomial coefficients, the figure is generated by averaging $5000$ trials of wGn signals.

Fig. 2, it is shown that $\beta$ decreases as $N$ increases with an approximate slope denoted by $S$, where $S = -0.0125$.

The empirical relationship between $N$ and $\beta$ can be modeled by a linear fit:

$$\beta(N) = S \log_2(N) + \beta(0) \tag{8}$$

where $\beta(0)$ is the y-intercept of the linear fit.

Based on eq. (8), eq. (5) can be re-written as follows:

$$\hat{\mathbb{W}}_i = \big(\beta(N) \times \mathbb{W}_1\big) \times \mathbf{2}^{-i} \quad i \geq 2 \tag{9}$$

From eq. (9), the estimated noise model will be a function of $N$ and the first IMF energy.

Subsequently, the energy model with a confidence interval that is based on the noise-only model ($\hat{\mathbb{W}}_i$) in either (5), (6), or (9) can be written as follows:

$$\log_2 \mathbb{T}_i = 2^{\mathbf{a}i+\mathbf{b}} + \log_2 \hat{\mathbb{W}}_i \quad i \geq 2 \tag{10}$$

where $\mathbb{T}_i$ is the $i^{th}$ IMF energy at a given confidence interval ($\alpha$), $\mathbf{a}$ and $\mathbf{b}$ are the fitting polynomial parameters for the noise-only model (5) where these parameters are given in [14, Table 3.1] for both $\alpha$ = 95% and 99%.

The energy model in eq. (10) is used as an adaptive threshold to discriminate the IMFs that have a high signal contribution from the ones of noise-only contributions. In that sense, and for spectrum sensing purposes, the designed probability of false alarm of the proposed energy detector is denoted by $P_{fa} = 1-\alpha$. On the other hand, the probability of detection (denoted by $P_d$) is the probability that at least one of the IMF energies in eq. (3) exceeds the energy model in eq. (10) such as:

$$P_d = Prob\left(\exists \left(\log_2 \mathbb{W}_i > \log_2 \mathbb{T}_i\right)\right), \quad i \geq 2 \tag{11}$$

Algorithm-1 describes the EMD-based energy detector scheme at the designed $P_{fa}$ for the $j^{th}$ channel.

**Algorithm 1**

**Input:** $\mathcal{M}_i(n) \in y(n) \leftarrow i^{th}$ IMF
**Output:** $\mathbb{S}_j \in \{\mathcal{H}_0, \mathcal{H}_1\} \leftarrow j^{th}$ channel status
**for** $j^{th}$ sensed channel **do**
  **for** $i := 1 \rightarrow K-1$ **do**
    $\mathbb{W}_i \leftarrow i^{th}$ IMF $energy\ metric\ of\ y(n)$
    $P_{fa} = 1 - \alpha \leftarrow$ The designed $P_{fa}$
    $\hat{\mathbb{W}}_i \leftarrow i^{th}$ IMF $energy\ of\ noise\ model$
    **if** $\exists(\log_2 \mathbb{W}_i > \log_2 \mathbb{T}_i)$ **then**
      $\mathbb{S}_j \in \mathcal{H}_1$
    **else**
      $\mathbb{S}_j \in \mathcal{H}_0$
    **end**
  **end**
**end**

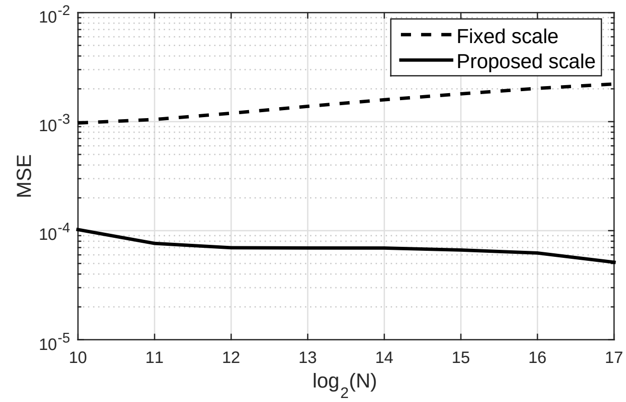



Figure 3: shows the mean squared error of both fixed scale and proposed scale with respect to known noise model using wGn signals.

## IV. SIMULATION RESULTS

In this section, single channel detection is performed to determine occupancy for different SNR and sample size scenarios. A baseband OFDM modulated signal with an observation period (T) is used with different sampling sizes; $N = 1000, 2000$, and $4000$. The channel is assumed to be flat and additive white Gaussian noise level is varied base on a range of SNR values. In this section, Monte Carlo simulations are carried out, where all results are the average of 5000 trials.

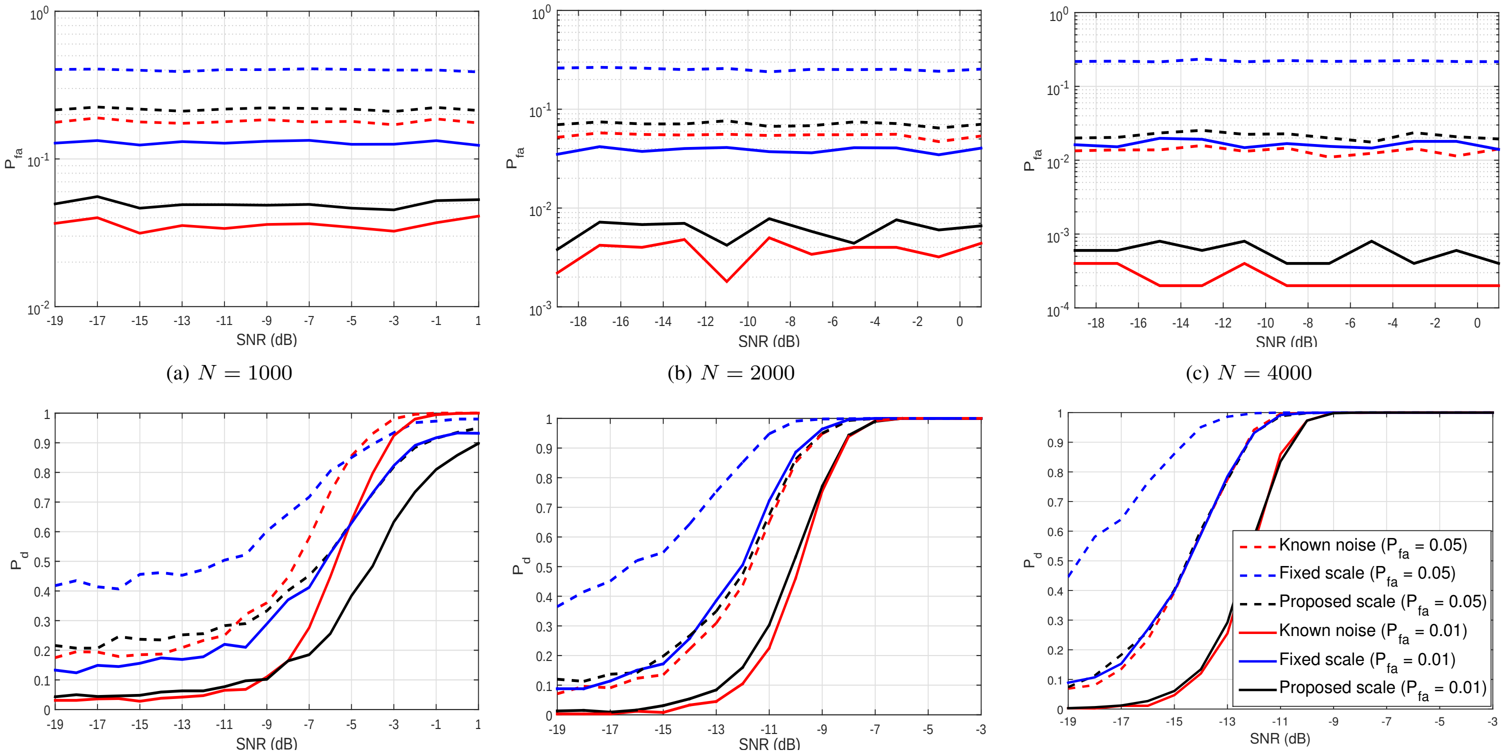
Three different energy models, known noise model eq. (5), fixed scale model eq. (6), and proposed adaptive scale model eq. (9) are used to evaluate the model in (10) in which these models have significant effect on the overall performance of the proposed detector. The known noise model refers to the assumption that the noise is known *a priori*, and that is not valid practically, but used here for demonstration purposes. The fixed scale model refers to the fact that the energy of the first IMF is scaled by a fixed quantity ($\sqrt{\rho}$).

In order to compare the proposed scale model versus the fixed scale model, the Mean Squared Error (MSE) between these models and the known noise is compared over different sample sizes ($N$). Fig. 3 illustrates the MSE of both fixed and proposed scale models in reference to the known noise model. From that figure, it is obvious that the proposed scale is more comparable to the known noise than the fixed scale model. The justification of that is due to the adaptive scaling of the first IMF energy (function of sample size) that results in better modelling for the total noise energy estimation of the received signal. However, the use of a fixed scale model ($\sqrt{\rho}$) yields in a misestimation of the noise energy and hence more deviation from the known noise model.

Next, the performance of the EMD-based energy detector is examined for each of the models (known noise, fixed scale, proposed scale) for different sample size and SNR values. The EMD-based detector is designed for two different probability of false alarms, $P_{fa}(designed) = 0.05$ for $\alpha = \%95$, and $P_{fa}(designed) = 0.01$ for $\alpha = \%99$. In Fig. (4a-4c), the actual probability of false alarm of the proposed EMD-based detector is obtained for different sample sizes, whereas increasing the sample size (from N = 1000 to 2000) has a significant effect on improving the false alarm rate. From Fig. (4a-4c), the proposed scale model performs closely to the known noise model, while the fixed scale model requires higher sample size (N = 4000) to satisfy the designed false alarm rate for $P_{fa}(designed) = \%1$.

On the other hand, Fig. (4d-4f) illustrates the detector performance (for $\%5$ and $\%1$ designed $P_{fa}$) in terms of probability of detection for different scenarios. The $P_d$ of the proposed scale model performs tightly to the known noise model and that performance is enhanced as the sample size increases. However, it is shown that the fixed scale model performs better than the other two models but that is jeopardized to its high false alarm rate exhibited in Fig. (4a-4c). From Fig. (4), the proposed scale model shows higher ability to estimate the noise of the received signal than the fixed scale model, and hence reveals better tradoff between the $P_d$ and $P_{fa}$.

Finally, the proposed scale model is selected to build an EMD-based energy detector and compared to other known techniques such as the Energy detector model (ED) in [15], and Maximum-Minimum Eigenvalue detector (MME) [16]. In this comparison, the designed $P_{fa}$ for all techniques is set to $\%1$. Both MME and the proposed method are blind and adaptive in terms of setting the detection threshold, unlike the ED which is sensitive to noise uncertainty [17]. Furthermore, both MME and the proposed method requires no prior knowledge about the noise energy, where MME is independent in its decision of noise energy, while the proposed method estimates it adaptively. In Fig. 5, the comparison between the aforementioned techniques shows that ED performs better than all other techniques. However, in the presence of noise uncertainty (1 dB), the performance of ED deteriorates significantly. The proposed method outperforms both MME (with a smoothing factor = 8) and the ED with noise uncertainty by a gain of 2 and 3 dB respectively to reach $P_d$ of $\%100$.




(a) $N = 1000$ (b) $N = 2000$ (c) $N = 4000$

(d) $N = 1000$ (e) $N = 2000$ (f) $N = 4000$

Figure 4: illustrates the probability of false alarm and the probability of detection for different SNR values and sample sizes for the known noise, fixed, and proposed scale energy models.

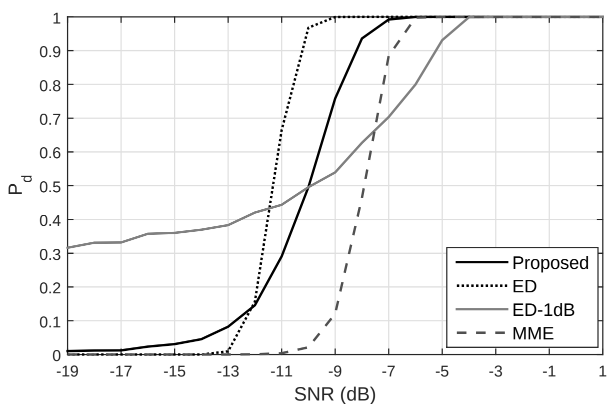



Figure 5: A comparison of the probability of detection of the proposed method versus ED and MME for $N = 2000$

## V. Conclusion

In this paper, an adaptive EMD-based energy detection scheme was presented for spectrum sensing in cognitive radio applications. This method exploits the ability of EMD to decompose a signal into IMFs in which the energy of these modes can be utilized for channel detection. The energy of the first IMF is scaled to evaluate the noise-only model which is used to obtain a detection threshold at some confidence interval ($\alpha$). The proposed scaling model exhibits better performance than the fixed scale model in terms of $P_d$ and $P_{fa}$. Further, the EMD-based energy detector (built upon the use of the proposed scale model) outperforms both ED (with a noise uncertainity) and MME over a range of SNR values.

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